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# Democratic mass matrices induced by strong gauge dynamics and large mixing angles for leptons

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## Abstract

We consider dynamical realization of the democratic type Yukawa coupling matrices as the Pendelton-Ross infrared fixed points. Such fixed points of the Yukawa couplings become possible by introducing many Higgs fields, which are made superheavy but one massless mode. Explicitly, we consider a strongly coupled GUT based on $SU(5) \times SU(5)$, where rapid convergence to the infrared fixed point generates sufficiently large mass hierarchy for quarks and leptons. Especially, it is found that the remarkable difference between mixing angles in the quark and lepton sectors may be explained as a simple dynamical consequence. We also discuss a possible scenario leading to the realistic mass spectra and mixing angles for quarks and leptons. In this scheme, the Yukawa couplings not only for top but also for bottom appear close to their quasi-fixed points at low energy and, therefore, $\tan\beta$ should be large.

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# 1 Introduction

Yukawa couplings are not direct observables, since they change depending on the choice of flavor basis. What we know in principle are the hierarchical masses of quarks and leptons, and the mixing matrices. The recent neutrino oscillation experiments [1, 2, 3, 4] indicate strongly that neutrinos are massive and lepton flavors are mixed. It is remarkable that the mixing angles ($\sin^2 2\theta_{\rm sun} \sim 0.84$ and $\sin^2 2\theta_{\rm atm} \sim 1.0$) are as large as nearly bi-maximal, which are in sharp contrast with the small mixing angles among quarks.

There have been proposed various phenomenological models explaining origin of the Yukawa couplings accommodating these features by assuming certain flavor symmetries. [5]. The most popular one would be the Froggatt-Nielsen mechanism [6] based on abelian flavor symmetries. In this line of thought, hierarchical structures among the Yukawa couplings are nicely explained. However the abelian symmetry itself cannot fix O(1) ambiguity of parameters. Therefore structure of the mixing angles in the lepton sector as well as the neutrino masses are explained no more than that those parameters are not hierarchical. Also the flavor differences are just input in the abelian flavor charges apriori.

Contrary to this, non-abelian flavor symmetries can impose firm relations among some elements of the Yukawa matrices. Therefore various non-abelian symmetries have been considered to explain the mixing angles in the lepton sector as well as the hierarchical masses simultaneously [7]. The democratic Yukawa matrices have been an attractive possibility of this kind [8, 9, 10, 11, 12]. The rigid democratic matrix, in which all elements are identical [4] , may be explained by assuming *e.g.* $S_3$ flavor symmetries for three generations of quarks and leptons. It is amusing that one of the mass eigenvalues appears much larger than others, though there is no apriori difference among flavors. The $S_3$ flavor symmetry is supposed to be broken slightly to give rise to small masses for the first and second generations. It has been known for some time that Fritzsch type mass matrices for quarks are realized after introducing a simple form of $S_3$ breaking parameters [8]. Moreover the lepton sector mixing angles uncovered by neutrino oscillation experiments may be explained in the democratic framework rather well [9, 10].

However no comprehensive explanation has been given to the origin of the flavor symmetry breaking terms in this framework. Moreover the $S_3$ flavor symmetry allows the mass matrix proportional to an identity matrix other than the democratic type matrix for the neutrino sector. The mixing matrix for the lepton sector can be obtained, only if we assume the neutrino mass matrix to be almost diagonal. Namely the democratic part must be small with some other reasons than the flavor symmetry [9, 10]. The same problem remains in the see-saw context, where both of Yukawa couplings and mass matrix of the right-handed neutrinos should be almost diagonal.

On the other hand, we face with a more stringent problem in considering the grand unified theories. In *e.g.* the SU(5) GUT, the matters belong to a $\mathbf{10}$ and a $\bar{\mathbf{5}}$. However $S_3$ symmetry allows an identity matrix for the Yukawa couplings among $\mathbf{10}$'s, which must be strictly forbidden for the quark mass hierarchy [11]. Thus the phenomenologically

[4] Phenomenologically the so-called extended democratic mass matrices [13] are equally considerable. However we restrict ourselves to the rigid cases in this paper.

postulated forms for the democratic Yukawa matrices are not explained only by flavor symmetries.

Indeed it is not a unique way to introduce some flavor symmetries and their small breakings in order to explain hierarchy among various couplings. For example, large mass hierarchy may be generated by overlapping of the wave functions in extra dimensions. Renormalization with large anomalous dimensions induced by strong dynamics also can generate mass hierarchy [14]. Similarly power-law running of the Yukawa couplings in the extra dimensions has been also utilized [15].

In this paper we consider the models in which the democratic type of Yukawa matrices are realized as infrared attractive fixed point couplings [16, 17, 18, 15]. It has been known for some time that ratio of the Yukawa couplings and the gauge coupling approach rapidly to the so-called Pendelton-Ross fixed points (PRFP) towards infrared [16]. If all elements of the Yukawa coupling matrix have an identical PRFP, then the democratic type matrix may be achieved dynamically. In order to make this possible, we need to introduce many Higgs fields. Also a special kind of mass terms for the multi Higgs fields are assumed so that they obtain masses of the GUT scale except for one massless mode, which is identified with Higgs in the low energy theory.

Indeed this idea is not new. Abel and King[18] have already considered several years ago except for the neutrino sector. In this paper, however, we stress that this mechanism is free from the difficulty in realizing democratic Yukawa matrices by the flavor symmetry. Moreover we are going to show that the typical difference in the mixing angles between quark and lepton sectors may be explained as a simple dynamical consequence in this framework. Our mechanism predicts very small neutrino Yukawa couplings, and therefore relatively light right handed neutrinos in the see-saw models.

Explicitly, we treat a supersymmetric GUT model based on $SU(5) \times SU(5)$ gauge group, in which the doublet-triplet problem is solved nicely [19, 20]. There one $SU(5)$ gauge interaction coupled with quarks and leptons is assumed to be strong and another is weak just as in the conventional SU(5) GUT. Then renormalization group (RG) from the Planck scale to the GUT scale shows that the Yukawa couplings in the GUT model are aligned in a very good accuracy due to strong dynamics. After diagonalizing the Yukawa matrix, only one coupling is found to be much larger than others. Further we will show that the realistic Yukawa coupling matrices may be obtained within a simple setup.

In this kind of scenarios, all of the Yukawa couplings, except for the neutrino sector, are given to be quite large at high energy. Because the Yukawa coupling and the gauge coupling are of the same order at the PRFP. However heavy top quark mass (178GeV) indicates that top Yukawa coupling is close to the so-called quasi-fixed point [21]. This means that top Yukawa coupling can be fairly large at the GUT scale, which is a favorable point of our scenario. However Yukawa couplings not only of top quark but also of bottom quark and tau lepton are all large. Therefore large $\tan\beta$ is predicted in this kind of scenarios.

The article is organized as follows. In section 2 we give a brief summary of the democratic type of mass matrices and the $S_3$ flavor symmetry. In section 3 we present the general ideas leading to the democratic Yukawa matrices at low energy by strongly

coupled gauge dynamics. There also the superpotential for the multi Higgs fields and their mass spectra are discussed. In section 4 we give an explicit model based on the $SU(5) \times SU(5)$ GUT. Then it is seen that sufficiently large hierarchy in couplings can be generated by RG from the Planck scale to the GUT scale. We also discuss predictions of this kind of scenarios and consistency with particle masses. There we work out the Yukawa couplings for top, bottom and tau obtained at low energy scale in the explicit model. The neutrino Yukawa couplings are discussed in section 5. There it is shown that the large mixing angles among leptons may be realized as a dynamical consequence, if the right-handed neutrinos have new Yukawa couplings to some other fields. In section 6 we consider minute structure of the democratic mass matrices producing realistic masses and mixing angles. Finally we devote section 7 to conclusions and discussions including comments on the extra dimensional setup and also on the soft supersymmetry breaking parameters.

## 2 Democratic mass matrices and $S_3$ flavor symmetry

In this section we give a brief review of the democratic mass matrices for quarks and leptons from the view point of $S_3$ flavor symmetry. The democratic mass matrices give a phenomenologically successful description. However it is found to be necessary to constrain the parameters allowed by the symmetry further. Namely the $S_3$ symmetry is not sufficient to give viable mass matrices. Indeed some attempts have been done in order to give comprehensive grounds to the viable democratic mass matrices so far [12].

The rigid democratic matrix $J$ is diagonalized as

$$J = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} = A \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} A^T, \tag{1}$$

by the diagonalization matrix

$$A = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -2/\sqrt{6} & 1/\sqrt{3} \end{pmatrix}. \tag{2}$$

For the democratic mass matrix, two vanishing eigenvalues are regarded as masses for the first and the second generations in the first approximation. Their masses are given by small deviations from the rigid matrix. It would be said that the interesting feature of this approach is the idea of flavor democracy, namely that there is no difference between flavors apriori.

In the standard model (SM), the democratic mass matrices are realized by assuming $S_3$ flavor symmetry in each kind of the SM matter fields. Hereafter we consider the minimal supersymmetric case, the MSSM. The superpotential of the MSSM is given as

$$W = Y^u_{ij}\ Q_i u_j H^u + Y^d_{ij}\ Q_i d_j H^d + Y^e_{ij}\ L_i e_j H^d + \frac{\kappa_{ij}}{2M_R}\ L_i L_j H^u H^u, \tag{3}$$

where $i, j = (1, 2, 3)$ represent generations. Each of $Q_i, u_i, d_i, L_i, e_i$ is assigned to a three dimensional representation of a distinct $S_3$. Then the Lagrangian is invariant under permutation of any set of the matter fields.

Let us start with mass matrices for quarks. It has been known that the realistic mass matrices are obtained by introducing small breaking parameters $\epsilon_{u(d)} \ll \delta_{u(d)} \ll 1$ of the flavor symmetry as follows [8].

$$
\begin{aligned}
M_q &\propto J + \begin{pmatrix} -\epsilon_q & 0 & 0 \\ 0 & \epsilon_q & 0 \\ 0 & 0 & \delta_q \end{pmatrix} \\
&= A \begin{pmatrix} 0 & -\sqrt{1/3}\epsilon_q & -\sqrt{2/3}\epsilon_q \\ -\sqrt{1/3}\epsilon_q & (2/3)\delta_q & -(\sqrt{2}/3)\delta_q \\ -\sqrt{2/3}\epsilon_q & -(\sqrt{2}/3)\delta_q & 1+(1/3)\delta_q \end{pmatrix} A^T,
\end{aligned}
\tag{4}
$$

where $q = u, d$. The matrix in the 2nd line is known as the Fritzsch type, which offers us a phenomenologically good representation.

The mass eigenvalues of these matrices are found to be

$$
m_1^q : m_2^q : m_3^q \sim \frac{\epsilon_q^2}{\delta_q} : \delta_q : 1. \tag{5}
$$

Therefore ratio of quark masses can be represented by assuming the breaking parameters of the following order;

$$
\begin{aligned}
&\epsilon_u \sim 10^{-4}, \quad \delta_u \sim 10^{-2}, \\
&\epsilon_d \sim 10^{-2}, \quad \delta_d \sim 10^{-1}.
\end{aligned}
\tag{6}
$$

The absolute values of the top and bottom quark masses are not determined by the flavor symmetry and, therefore, must be parameterized phenomenologically. In our dynamical mechanism, we can predict these absolute values by evaluating the Yukawa couplings at low energy. This will be discussed later.

The mixing angles turn out to be small, since the diagonalization matrix for $M_u$ and $M_d$ are both close to $A$. Quantitatively the CKM matrix is given in a good approximation by

$$
U_{\mathrm{CKM}} \sim \begin{pmatrix} 1 & O(\epsilon_d/\delta_d) & O(\epsilon_d) \\ O(\epsilon_d/\delta_d) & 1 & O(\delta_d) \\ O(\epsilon_d) & O(\delta_d) & 1 \end{pmatrix}. \tag{7}
$$

It is found that the above choice of parameters may reproduce the realistic mixing angles also.

Next let us go into the lepton sector. The mass matrix for the charged leptons are given in a similar fashion;

$$
M_e \propto J + \begin{pmatrix} -\epsilon_e & 0 & 0 \\ 0 & \epsilon_e & 0 \\ 0 & 0 & \delta_e \end{pmatrix}. \tag{8}
$$

Here, if we take the flavor symmetry breaking parameters as

$$\epsilon_e \sim 10^{-2}, \quad \delta_e \sim 10^{-1}, \tag{9}$$

then ratio of the charged lepton masses are parameterized well. Again the absolute value is undetermined in this approach, though it will be determined in our scheme.

However situation for the neutrino mass matrix is somewhat different. The $S_3$ flavor symmetry allow the couplings $\kappa_{ij}$ given by Eq. (3) proportional to an identity also. Therefore the mass matrix is given as

$$M_\nu \propto \ I \ + \ rJ + \begin{pmatrix} 0 & 0 & 0 \\ 0 & \epsilon_\nu & 0 \\ 0 & 0 & \delta_\nu \end{pmatrix}, \tag{10}$$

where $I$ denotes an identity matrix.

It is noted that this mass matrix may explain the mixing angles in the lepton sector nicely, if the parameter $r$ in Eq. (10) is much smaller than 1. For vanishing $r$, the lepton mixing matrix (or the MNS matrix) is just given by $A^T$ (matrix $A$ is given explicitly in Eq. (2)). Therefore we obtain the three mixing angles as

$$\sin^2 2\theta_{12} \sim 1, \quad \sin^2 2\theta_{23} \sim 0.94, \quad U_{e3} = 0. \tag{11}$$

This should be compared with observation by the recent neutrino experiments [1, 2, 3, 4], which tells us

$$\sin^2 2\theta_{12} \sim 0.84, \quad \sin^2 2\theta_{23} \sim 1.0, \quad |U_{e3}| < 0.23. \tag{12}$$

Thus the lepton mixing angles are found to be almost explained already without parameter $r$ in Eq. (10). In other words, a nearly diagonal neutrino mass matrix is favorable including the flavor symmetry breaking part from phenomenological point of view. However we do not find any reasons either why $r$ is so small, or why the flavor symmetry breaking parameters are input only at diagonal elements.

This fine-tuning problem remains or becomes more curious in the see-saw mechanism. The superpotential for the neutrino sector is given by

$$W = Y^\nu_{ij} L_i \nu_j H^u \ + \ \frac{1}{2} M_{Rij} \nu_i \nu_j, \tag{13}$$

where $\nu_i$ denote the right-handed neutrinos. Here we need to assume that these right-handed neutrinos belong to the same representation of $S_3$ as the lepton doublets $L_i$. If we assume a distinct $S_3$ group for the right-handed neutrinos like other fields, the neutrino Yukawa coupling matrix $Y^\nu$ is restricted to the democratic form. Then the mixing angles cannot be large. The neutrino Yukawa matrix and the right-handed neutrino mass matrix are now parameterized as

$$\begin{aligned} Y^\nu &= y^\nu_0 (I + rJ) \ + \ \Delta Y^\nu, && (14) \\ M_R &= M_{R0} (I + r'J) \ + \ \Delta M_R. && (15) \end{aligned}$$

In should be noted that both of $Y^\nu$ and $M_R$ should be nearly diagonal in order to have large mixing angles . Therefore the free parameters $r$ and $r'$ are constrained to be small, which is not explained by the flavor symmetry.

Moreover this sort of fine-tuning turns out to be required much stronger in the GUT models. In the SU(5) GUT, the quark and lepton fields in one generation are combined into a $\mathbf{10}+\bar{\mathbf{5}}+\mathbf{1}$ representation. The superpotential of their Yukawa interactions is given by

$$W = Y^u_{ij}\ \mathbf{10}_i\mathbf{10}_j H(\mathbf{5}) + Y^d_{ij}\ \mathbf{10}_i\mathbf{5}^*{}_j H(\mathbf{5}^*) + Y^\nu_{ij}\ \mathbf{5}^*{}_i\mathbf{1}_j H(\mathbf{5}). \tag{16}$$

We note that Yukawa couplings satisfy $Y^e = (Y^d)^T$ at the GUT scale. Then $S_3$ symmetry for $\mathbf{10}_i$ allows the Yukawa coupling matrix of $Y^u = y^u_0(J + r''I)$ in general. However the mass hierarchy of up-sector quarks cannot be obtained unless $r''$ is suppressed to $O(10^{-5})$.

Thus the approach of democratic mass matrix is an attractive possibility phenomenologically, but the flavor symmetries do not support origin of the assumed Yukawa couplings. In the next section, we consider realization of the democratic Yukawa matrices in a dynamical fashion without recourse to any flavor symmetries. There it will be seen that we are not troubled with the fine-tuning problems any more.

# 3 Flavor democracy by strong gauge dynamics

Our basic assumption is that the Yukawa couplings are not hierarchical nor aligned, but may be even somewhat anarchy at the fundamental scale. The purpose of this section is to show that the Yukawa couplings can be aligned to the rigid democratic form quite minutely at lower energy scale owing to strong gauge dynamics. The realization of realistic matrices are discussed in the later section. Also we consider such a mechanism in GUT models, since the problem of flavor symmetry is very severe especially in GUT.

## 3.1 IR fixed point for a single Yukawa coupling

Before going into the democratic Yukawa matrix, let us consider the IR fixed point a la Pendelton and Ross [16] in the cases with a single Yukawa coupling. In general, the RG equations for the gauge coupling $g$ and the Yukawa coupling $y$ are given at one-loop level as

$$\mu\frac{d\alpha_g}{d\mu} = -b{\alpha_g}^2, \tag{17}$$

$$\mu\frac{d\alpha_y}{d\mu} = (a\alpha_y - c\alpha_g)\alpha_y, \tag{18}$$

where we defined $\alpha_g = g^2/8\pi^2$ and $\alpha_y = |y|^2/8\pi^2$. The coefficients depend on the fields contents and the Yukawa interaction, but note that $a$ and $c$ are positive constants. Then the ratio $x = \alpha_y/\alpha_g$ satisfies the RG equation,

$$\mu\frac{dx}{d\mu} = [ax - (c-b)]\,\alpha_g x. \tag{19}$$

This beta function has a non-trivial fixed point at $x = x^* = (c-b)/a$, which is IR attractive.

The convergence behavior of the RG flows around the fixed point may be seen by linear analysis. The deviation from the fixed point, $\Delta x = x - x^*$, is subject to the equation given by

$$\mu \frac{d\Delta x}{d\mu} = (c-b)\alpha_g \Delta x. \tag{20}$$

If the gauge coupling does not change rapidly, then the flows of $x$ around the IR fixed point may be evaluated as

$$\Delta x(\mu) \sim \left(\frac{\mu}{\Lambda}\right)^{(c-b)\alpha_g} \Delta x(\Lambda). \tag{21}$$

We see that the conditions for strong convergence as follows; (i) The gauge coupling is large. (ii) The coefficient $(c-b)$ is positive. Also larger $(c-b)$ makes convergence stronger. The second condition means that the coefficient $b$ should not be very large, therefore, that asymptotically free gauge theories with rapidly running gauge couplings are excluded. [5]

Such a fixed point appears in gauge theories beyond four space-time dimensions as well[15]. In the dimensions of $4+\delta$, the RG equations for the gauge coupling and the Yukawa coupling at one-loop are given as

$$\begin{aligned} \mu \frac{d\alpha_g}{d\mu} &= \delta\alpha_g - b{\alpha_g}^2, & (22)\\ \mu \frac{d\alpha_y}{d\mu} &= \delta\alpha_y + (a\alpha_y - c\alpha_g)\alpha_y, & (23) \end{aligned}$$

where $\alpha_g$ and $\alpha_y$ denote dimensionless couplings. Then the RG equation for $x = \alpha_y/\alpha_g$ is found to be identical to Eq. (19). We should note some particular features in the extra-dimensional theories. First the running law of these couplings is not logarithmic but power with respect to the renormalization scale. Also the gauge coupling becomes strong at ultraviolet irrespectively of the sign of $b$. With these reasons, convergence to the IR fixed point is found to be very strong in general.

## 3.2 Models with 3 flavors

If we extend the Yukawa interaction to multi-flavors naively, then the IR fixed points for the Yukawa matrices may exist but are proportional to an identity matrix. Thus the democratic Yukawa coupling matrix cannot be obtained by renormalization effect in MSSM. However it turns out to be possible once we admit multi Higgs fields.

First let us consider only the up-type Yukawa couplings in the superpotential given by Eq. (3) or by Eq. (16). We introduce 9 elementary higgs superfields $H_{ij}(i,j=1,2,3)$

[5]In asymptotically non-free gauge theories, it is easier to have strong convergence, since the gauge couplings become large at high energy and $b$ is negative[22].

with the same properties, and extend the Yukawa interactions as

$$W = \sum_{i,j=1,2,3} Y_{ij} Q_i u_j H_{ij}. \tag{24}$$

In the SU(5) GUT, we may take $Q = u = \mathbf{10}$ and H=H($\mathbf{5}$). In the supersymmetric theories the beta functions for the Yukawa couplings $Y_{ij}$ are written down in terms of the anomalous dimensions of $Q_i$, $u_i$ and $H_{ij}$. This is due to the so-called non-renormalization of superpotentials. Explicitly these anomalous dimensions may be written down as

$$\gamma_{Q_i} = \left[ a_Q(\alpha_{y_{i1}} + \alpha_{y_{i2}} + \alpha_{y_{i3}}) - c_Q \alpha_g \right], \tag{25}$$

$$\gamma_{u_i} = \left[ a_u(\alpha_{y_{1i}} + \alpha_{y_{2i}} + \alpha_{y_{3i}}) - c_u \alpha_g \right], \tag{26}$$

$$\gamma_{H_{ij}} = \left[ 3a_H \alpha_{y_{ij}} - c_H \alpha_g \right], \tag{27}$$

where we defined $\alpha_{y_{ij}} = |Y_{ij}|^2/(8\pi^2)$ as well. Note that the above interactions in Eq. (24) induce anomalous dimensions only in the diagonal elements, which are given above. Then the RG equation for $\alpha_{y_{ij}}$ is simply given by

$$\mu \frac{d\alpha_{y_{ij}}}{d\mu} = \left( \gamma_{Q_i} + \gamma_{u_j} + \gamma_{H_{ij}} \right) \alpha_{y_{ij}}. \tag{28}$$

It is easy to see existence of a non-trivial fixed point by using the beta function given by Eq. (28). The RG equations for $x_{ij} = \alpha_{y_{ij}}/\alpha_g$ are written down as

$$\mu \frac{dx_{ij}}{d\mu} = \left[ (b-c) + \left( \hat{\gamma}_{Q_i} + \hat{\gamma}_{u_j} + \hat{\gamma}_{H_{i,j}} \right) \right] \alpha_g x_{ij}, \tag{29}$$

where we have defined $c = c_Q + c_u + c_H$ and also

$$\hat{\gamma}_{Q_i} = a_Q(x_{i1} + x_{i2} + x_{i3}), \tag{30}$$

$$\hat{\gamma}_{u_i} = a_u(x_{1j} + x_{2j} + x_{3j}), \tag{31}$$

$$\hat{\gamma}_{H_{ij}} = 3a_H x_{ij}. \tag{32}$$

Therefore the condition for the non-trivial fixed point is that the combinations in the brace of the Eq. (29) vanish for all sets of $(i, j)$. The solution is unique and is found to be

$$x^*_{ij} = x^* = \frac{c-b}{3a}, \tag{33}$$

where $a = a_Q + a_u + a_H$.

The non-trivial but important matter is whether this fixed point is really IR attractive or not. This may be seen by linear analysis of the RG equations around the fixed point

(33). The deviations from the fixed point $\Delta x_{ij}$ satisfy the following equation;

$$\mu\frac{d\Delta x_{ij}}{d\mu}=\alpha_g x^*\begin{pmatrix} a' & a_Q & a_Q & a_u & 0 & 0 & a_u & 0 & 0 \\ a_Q & a' & a_Q & 0 & a_u & 0 & 0 & a_u & 0 \\ a_Q & a_Q & a' & 0 & 0 & a_u & 0 & 0 & a_u \\ a_u & 0 & 0 & a' & a_Q & a_Q & a_u & 0 & 0 \\ 0 & a_u & 0 & a_Q & a' & a_Q & 0 & a_u & 0 \\ 0 & 0 & a_u & a_Q & a_Q & a' & 0 & 0 & a_u \\ a_u & 0 & 0 & a_u & 0 & 0 & a' & a_Q & a_Q \\ 0 & a_u & 0 & 0 & a_u & 0 & a_Q & a' & a_Q \\ 0 & 0 & a_u & 0 & 0 & a_u & a_Q & a_Q & a' \end{pmatrix}\Delta x_{ij}, \tag{34}$$

where $a'=a_Q+a_u+3a_H$. The eigenvalues of this matrix are found to be $3a_H$, $3a_H$, $3a_H$, $3a_H$, $3(a_Q+a_H)$, $3(a_Q+a_H)$, $3(a_u+a_H)$, $3(a_u+a_H)$, $3a$. It should be noted that all of these are positive, which ensures IR attractiveness of the fixed point. Thus all Yukawa couplings may be aligned to the same value by renormalization effect. [6]

It is possible also to reduce the number of Higgs upto three by assuming a $Z_3$ symmetry as follows. We assign the $Z_3$-charge $q_i=2\pi/3i$ to $(Q_i,u_i,H_i)(i=1,2,3)$ respectively. Then the $Z_3$ symmetry restricts the Yukawa interactions to

$$W=\sum_{i,j=1,2,3}Y_{ij}Q_iu_jH_{3-i-j}, \tag{35}$$

where the indexes are defined modulo 3. With these interactions the anomalous dimensions are given to be diagonal. Explicitly they are found to be

$$\gamma_{Q_i} = \left[a_Q(\alpha_{y_{i1}}+\alpha_{y_{i2}}+\alpha_{y_{i3}})-c_Q\alpha_g\right], \tag{36}$$

$$\gamma_{u_i} = \left[a_u(\alpha_{y_{1i}}+\alpha_{y_{2i}}+\alpha_{y_{3i}})-c_u\alpha_g\right], \tag{37}$$

$$\gamma_{H_i} = \left[a_H(\alpha_{y_{1(2-i)}}+\alpha_{y_{2(1-i)}}+\alpha_{y_{3(3-i)}})-c_H\alpha_g\right]. \tag{38}$$

The RG equations for $x_{ij}=\alpha_{y_{ij}}/\alpha_g$ are given by

$$\mu\frac{dx_{ij}}{d\mu}=\left[(b-c)+\left(\hat\gamma_{Q_i}+\hat\gamma_{u_j}+\hat\gamma_{H(3-i-j)}\right)\right]\alpha_g x_{ij}, \tag{39}$$

where $c=c_Q+c_u+c_H$ and $\hat\gamma=\gamma/\alpha_g$ again. It is immediately seen that couplings $x^*_{ij}=x^*=(c-b)/3a$ $(a=a_Q+a_u+a_H)$ give a set of fixed point solutions. However it is found that this is not a unique solution. The linear perturbation around the fixed point

[6] To be precise, what is aligned at low energy is not a Yukawa coupling but it's absolute value. The complex phase is not controlled by the dynamics. In this article, we treat the Yukawa couplings as if real, and do not discuss the complex phases.

$\Delta x_{ij}$ satisfies the differential equation,

$$\mu \frac{d\Delta x_{ij}}{d\mu} = \alpha_g x^* \begin{pmatrix} a & a_Q & a_Q & a_u & 0 & a_H & a_u & a_H & 0 \\ a_Q & a & a_Q & a_H & a_u & 0 & 0 & a_u & a_H \\ a_Q & a_Q & a & 0 & a_H & a_u & a_H & 0 & a_u \\ a_u & a_H & 0 & a & a_Q & a_Q & a_u & 0 & a_H \\ 0 & a_u & a_H & a_Q & a & a_Q & a_H & a_u & 0 \\ a_H & 0 & a_u & a_Q & a_Q & a & 0 & a_H & a_u \\ a_u & 0 & a_H & a_u & a_H & 0 & a & a_Q & a_Q \\ a_H & a_u & 0 & 0 & a_u & a_H & a_Q & a & a_Q \\ 0 & a_H & a_u & a_H & 0 & a_u & a_Q & a_Q & a \end{pmatrix} \Delta x_{ij}, \tag{40}$$

where $a = a_Q + a_u + a_H$. The eigenvalues of this matrix are found to be 0, 0, $2a_Q$, $2a_Q$, $2a_u$, $2a_u$, $2a_H$, $2a_H$, $2a$. Existence of two zero mode, namely constant modes, means that the fixed point solution found above is not IR attractive. In other words, the Yukawa couplings obtained at low energy are dependent on their initial values, which are now supposed to be disordered.

However further flavor symmetry may relieve us of this problem. Let us assume *e.g.* a discrete symmetry among matters,

$$Q_i \to Q_{i+1}, \quad u_i \to u_{i-1}, \quad H_i \to H_i. \tag{41}$$

Then this symmetry imposes additional relations for the Yukawa couplings, which are given by

$$x_{ij} = x_{(i+1)(j-1)} = x_{(i-1)(j+1)}. \tag{42}$$

Then it is found that the zero modes in Eq. (40) are just forbidden by this discrete symmetry. Also linear perturbation shows us that the fixed point is restricted to a unique one and is IR attractive. Thus we have seen that the number of Higgs fields may be reduced by assuming some flavor structures in the fundamental theory. However we do not consider this possibility more in this article.

## 3.3 Flavor democratic Higgs

So far we have seen that the gauge dynamics may align the Yukawa couplings to the same value at IR, once many Higgs fields are introduced. This does not lead us to a democratic mass matrix immediately. Only one mode composed of many Higgs fields, which is identified with the Higgs field in the MSSM, should be massless and all others should be superheavy and decoupled at low energy. Moreover every fundamental Higgs field must contain the massless mode by the same amount, otherwise the Yukawa couplings are deformed from the democratic ones.

In practice, we may construct the mass terms for the Higgs fields which ensure the above properties [18]. We return to the 9 Higgs model given by Eq. (24). In the case of

SU(5) GUT, an example of the superpotential for the Higgs fields may be given by

$$
\begin{aligned}
W &= M\sum_i (H(\mathbf{5})_{(i+1)j} - H(\mathbf{5})_{ij})(H(\bar{\mathbf{5}})_{(i+1)j} - H(\bar{\mathbf{5}})_{ij}) \\
&\quad + M\sum_j (H(\mathbf{5})_{i(j+1)} - H(\mathbf{5})_{ij})(H(\bar{\mathbf{5}})_{i(j+1)} - H(\bar{\mathbf{5}})_{ij}). \qquad (43)
\end{aligned}
$$

Then the mass terms for the Higgs scalar fields,

$$
M^2\sum_i |H(\mathbf{5})_{(i+1)j} - H(\mathbf{5})_{ij}|^2 + M^2\sum_j |H(\mathbf{5})_{i(j+1)} - H(\mathbf{5})_{ij}|^2 + \cdots, \qquad (44)
$$

give rise to just one massless mode. Explicitly the massless mode $H$ is given by

$$
H = \frac{1}{3}\sum_{i,j} H_{ij}. \qquad (45)
$$

Presence of the massless mode is ensured by the shift symmetry with transformations [7],

$$
H(\mathbf{5})_{i,j} \to C, \quad H(\bar{\mathbf{5}})_{i,j} \to \bar{C}, \qquad (46)
$$

where $C$ and $\bar{C}$ are constants. Other modes acquire mass of order $M$, which gives decoupling scale. In the later section, we shall take $M$ to be the GUT scale.

This massless modes composed of $H(\mathbf{5})$ and $H(\bar{\mathbf{5}})$ are identified with Higgs fields $H^u$ and $H^d$ in the MSSM respectively. It is important to note that the Higgs field $H_{ij}$ in the both sector contains the zero mode $H$ with same factor

$$
H_{ij} = \frac{1}{3}H + \cdots, \qquad (47)
$$

for all $(i, j)$. Therefore the Yukawa matrix appearing in the MSSM is given to be democratic one, once the Yukawa couplings have been aligned enough by the strong gauge dynamics.

It may be said that the basic policy in our consideration is flavor democracy. We do not assume any hierarchical structures in the initial Yukawa couplings. In addition the superpotential for Higgs field does not have flavor difference. Therefore, there is no essential difference among flavors apriori.

# 4 A model based on the $SU(5) \times SU(5)$ GUT

Next we consider the mass hierarchy between the first two generations and the third generation. The small masses for the first and the second generations are generated by slight deviations from the rigid democratic mass matrix. From our standpoint of flavor democracy, these deviations should be traces of disorder in the initial couplings. Mass

[7] This structure is common to the deconstruction models[23], where the massless mode is generated as a Nambu-Goldstone boson.

ratio between up and top quarks is more than $1:10^5$. On the other hand, ratio between the GUT scale and the Planck scale, which we suppose to be the fundamental scale, is only about $1:10^3$. Therefore we need very strong gauge interaction in order to achieve such a hierarchy. We may estimate roughly strength of the gauge coupling by assuming that it is not running. It is seen from Eq. (21) that the gauge coupling must satisfies

$$\frac{g^2}{(4\pi)^2} > \frac{5}{6(c-b)}. \tag{48}$$

This condition implies that the gauge coupling need to be non-perturbatively large.

Naively the gauge coupling constant of the SU(5) GUT is supposed to be rather weak, since unification of the three gauge couplings in the MSSM occurs at rather weak coupling regime. Therefore it seems unlikely to make the gauge coupling of GUT so strong. Actually this is not the case. The gauge coupling unification is not destroyed in the presence of any extra heavy fields belonging to SU(5) multiplets. However corrections by the extra matters enlarge the unified gauge coupling. Thus one way to realize strong unification is to assume a suitable number of extra heavy fields in the MSSM [18].

Here we consider another scenario which may be more attractive [8]. Suppose that the gauge group is given by a product of two $SU(5)$ groups and is broken spontaneously to their diagonal subgroup at the unification scale. We represent the gauge couplings for the two groups $SU(5)'$ and $SU(5)''$ by $g'$ and $g''$ respectively. Here we assume that all of the matter fields as well as the Higgs fields are charged under $SU(5)'$ but singlet under $SU(5)''$ and, moreover, that that $g'$ is non-perturbatively strong but $g''$ is weak. Then RG for the Yukawa couplings is driven by the $SU(5)'$ gauge interaction, and may be attracted to their IR fixed point very rapidly. Also the gauge coupling $g$ for the diagonal subgroup, which is given by $1/g^2 = 1/g'^2 + 1/g''^2$, is given weak.

The superpotential to be considered here is the same as that given in Eq. (16) except for the Higgs fields extended to multi ones;

$$W = Y^u_{ij}\ \mathbf{10}_i\mathbf{10}_j H(\mathbf{5})_{ij} + Y^d_{ij}\ \mathbf{10}_i\bar{\mathbf{5}}_j H(\bar{\mathbf{5}})_{ij} + Y^\nu_{ij}\ \bar{\mathbf{5}}_i\mathbf{1}_j H(\mathbf{5})_{ij}. \tag{49}$$

To be explicit, the anomalous dimensions for the matter fields and the Higgs fields are given at one-loop level as follows;

$$\gamma_{\mathbf{10}_i} = -\frac{36}{5}\alpha_g{}' + 3\sum_{j=1}^3 \alpha_{y^u_{ij}} + 2\sum_{j=1}^3 \alpha_{y^d_{ij}}, \tag{50}$$

$$\gamma_{\bar{\mathbf{5}}_i} = -\frac{24}{5}\alpha_g{}' + 4\sum_{j=1}^3 \alpha_{y^d_{ji}} + \sum_{j=1}^3 \alpha_{y^\nu_{ij}}, \tag{51}$$

$$\gamma_{\mathbf{1}_i} = 5\sum_{j=1}^3 \alpha_{y^\nu_{ji}}, \tag{52}$$

[8]One may expect also that power law running of gauge coupling in extra dimensions offers us models with strong convergence as well. However we do not pursuit for this direction due to a problem discussed in section 7.

$$\gamma_{H(\mathbf{5})_{ij}} = -\frac{24}{5}{\alpha_g}' + 6{\alpha_y}^u_{ij} + {\alpha_y}^\nu_{ij}, \tag{53}$$
$$\gamma_{H(\bar{\mathbf{5}})_{ij}} = -\frac{24}{5}{\alpha_g}' + 4{\alpha_y}^d_{ij}, \tag{54}$$

where $\alpha_g = g'^2/8\pi^2$ and ${\alpha_y}^a_{ij} = |Y^a_{ij}|^2/8\pi^2$ for $a = u, d, \nu$. The RG equations for the Yukawa couplings $Y^u_{ij}, Y^d_{ij}$ and $Y^\nu_{ij}$ are written down in terms of these anomalous dimensions as

$$\mu\frac{d{\alpha_y}^u_{ij}}{d\mu} = \left(\gamma_{\mathbf{10}_i} + \gamma_{\mathbf{10}_j} + \gamma_{H(\mathbf{5})_{ij}}\right){\alpha_y}^u_{ij}, \tag{55}$$
$$\mu\frac{d{\alpha_y}^d_{ij}}{d\mu} = \left(\gamma_{\mathbf{10}_i} + \gamma_{\bar{\mathbf{5}}_j} + \gamma_{H(\bar{\mathbf{5}})_{ij}}\right){\alpha_y}^d_{ij}, \tag{56}$$
$$\mu\frac{d{\alpha_y}^\nu_{ij}}{d\mu} = \left(\gamma_{\bar{\mathbf{5}}_i} + \gamma_{\mathbf{1}_j} + \gamma_{H(\mathbf{5})_{ij}}\right){\alpha_y}^\nu_{ij}. \tag{57}$$

In the next section we consider a dynamical mechanism compatible with large mixing angles in the lepton sector. There all of the neutrino Yukawa couplings $Y^\nu_{ij}$ are made decrease to very small values. With this reason, we shall consider RG behavior of other Yukawa couplings by neglecting the neutrino Yukawa couplings in this section.

Now two kinds of Yukawa couplings ${\alpha_y}^u$ and ${\alpha_y}^d$ are coupled with each other in the RG equations. Regardless of this complication, it is found that a non-trivial fixed point exists and is IR attractive. The coupling ratios $x^u_{ij} = {\alpha_y}^u_{ij}/\alpha_g$ and $x^d_{ij} = {\alpha_y}^d_{ij}/\alpha_g$ satisfy the following equations,

$$\mu\frac{dx^u_{ij}}{\mu} = \left(-\frac{96}{5} + b + \sum_{k=1}^{3}\left(3x^u_{ik} + 3x^u_{jk} + 2x^d_{ik} + 3x^d_{jk}\right) + 6x^u_{ij}\right){\alpha_g}' x^u_{ij}, \tag{58}$$
$$\mu\frac{dx^d_{ij}}{\mu} = \left(-\frac{84}{5} + b + \sum_{k=1}^{3}\left(3x^u_{ik} + 2x^d_{ik} + 4x^d_{kj}\right) + 4x^d_{ij}\right){\alpha_g}' x^d_{ij}. \tag{59}$$

Then it is straightforward to find the non-trivial fixed point solution, which turns out to be

$$x^u_{ij} = x^{u*} = \frac{552 - 25b}{1050} \sim 0.53 - 0.02b, \tag{60}$$
$$x^d_{ij} = x^{d*} = \frac{384 - 25b}{700} \sim 0.55 - 0.04b. \tag{61}$$

Thus the Yukawa couplings are fixed at the GUT scale, once $b$ is given. In any case, the Yukawa couplings become non-perturbatively large accompanied with the gauge coupling. It is a rather tedious problem to verify the IR attractive nature of this fixed point by linear perturbation. However it is quite obvious to see it by solving the differential equations given by (58) and (59) numerically. In Fig. 1, the flow lines for $x^u_{ij}$ and $x^d_{ij}$ are shown in the case of $b = 0, {\alpha_g}' = 1.0$. Their initial conditions are chosen at random just for demonstration. It is seen that both couplings converge their fixed point values very rapidly.

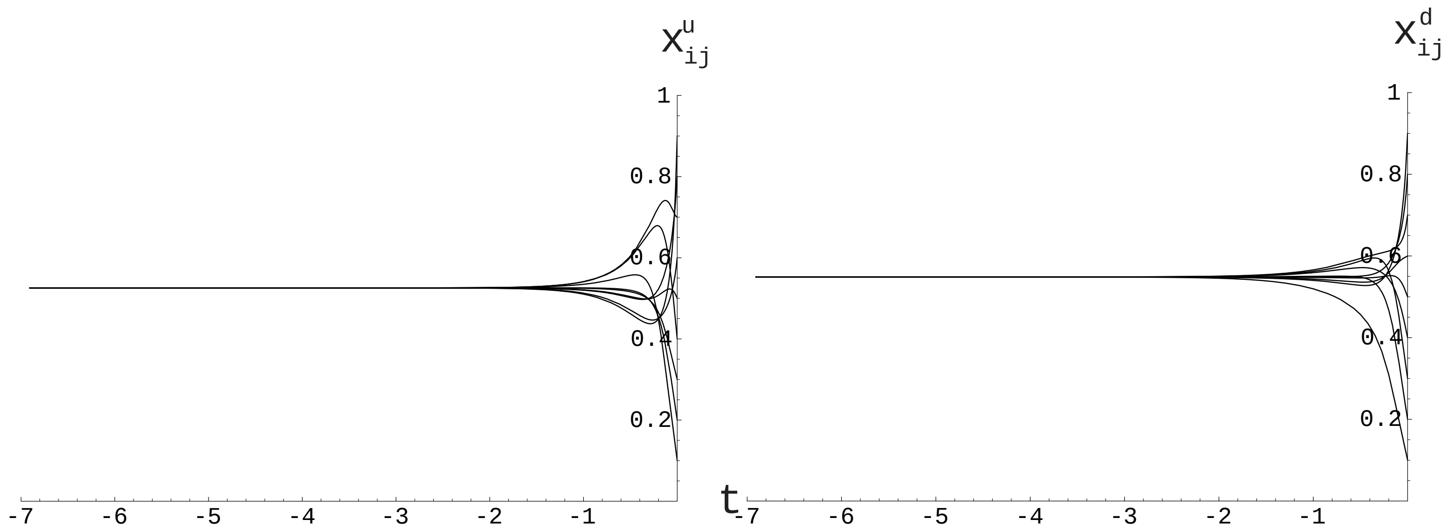


Figure 1: RG running of $x^u_{ij}$ and $x^d_{ij}$ for a sample of the initial couplings chosen at random. We assumed also $b = 0, {\alpha_g}' = 1.0$. The parameter $t$ represent the renormalization scale $\mu$ by using $\ln(\mu/M_{\rm pl})$, where $M_{\rm pl}$ is the Plank scale.

People have studied GUT models based on a product group for the purpose of avoiding the doublet-triplet splitting problem [19, 20]. Recently models based on $G = SU(5)' \times SU(5)''$ also have been proposed in this context [20], where gauge charge assignment for Higgs fields is different from the above setting. It is assumed that $H(\bar{\mathbf{5}})$ carries $(\mathbf{1}, \bar{\mathbf{5}})$ charges for the product group $SU(5)' \times SU(5)''$, while $H(\bar{\mathbf{5}})$ carries $(\mathbf{5}, \mathbf{1})$ as before. It is noted that the down-type Yukawa interactions in the superpotential given by Eq. (49) are not invariant any more. In order to generate the down-type Yukawa interactions, we introduce another field $\Sigma$ belonging to $(\bar{\mathbf{5}}, \mathbf{5})$. We suppose also that the spontaneous symmetry breaking $SU(5)' \times SU(5)'' \to SU(5)_{\rm diag}$ is induced by a vacuum expectation value of $\Sigma$. Then an invariant, but non-renormalizable, term

$$\frac{Y^d_{ij}}{\Lambda}\ \mathbf{10}_i \bar{\mathbf{5}}_j H(\bar{\mathbf{5}})_{ij} \Sigma \tag{62}$$

may generate the down-type Yukawa couplings effectively after symmetry breaking. It seems rather difficult to discuss renormalization involved with non-renormalizable operators like this. However we may also consider the corrections at the broken vacuum, where these operators plays a role of the Yukawa interactions instead. Therefore it is expected that the above analysis for the IR fixed point is equally applied to this model.

As a general property of the present mechanism, the Yukawa couplings for the third generations are given to be rather large at the GUT scale. The masses for top, bottom and tau are given in terms of the Yukawa couplings obtained after diagonalization. It is easy to find these Yukawa couplings renormalized at $O(100)$GeV by solving RG equations

for the MSSM. The neutrino Yukawa couplings may well be neglected in this analysis. Then the low energy couplings are determined with respect to the strong gauge coupling $g'$ and the model parameter $b$. However it is found that the resultant Yukawa couplings are almost insensitive to these parameters. This is because the initial Yukawa couplings are fairly large and such flows converge to the so-called quasi-fixed point [21]. To be explicit, the low energy couplings are found to be $Y_t = 1.00$, $Y_b = 0.94$ and $y_\tau = 0.62$ in the case of $\alpha_g{}' = g'^2/8\pi^2 = 1.0$ and $b = 0$. It is seen first that $\tan\beta$ should be large as $O(50)$, to explain the mass ratio $m_t/m_b$. since $Y_t$ and $Y_b$ appear to be almost the same. Therefore mass of top quark is predicted to be the same as the vacuum expectation value of the neutral Higgs 174GeV, which shows a very good agreement with observation. We may also predict mass ratio of bottom and tau, $m_b/m_\tau$. The above couplings bring us to $m_b/m_\tau = 1.52$, which is slightly smaller than the expected value.[9]

# 5 Neutrino sector

In the previous section we have seen that the Yukawa couplings are aligned to the democratic forms very well. The deviations from the rigid democratic form are responsible for masses of the first two generations and mixing angles among quarks. However the mixing matrix becomes necessarily close to an identity matrix for any small deviations. If we take the neutrino Yukawa couplings, ignored in the previous section, into account, then these couplings are found to be attracted to an IR fixed point by strong dynamics as well as other Yukawa couplings. Obviously this is incompatible with the observed large mixing angles for leptons.

In this respect, it is phenomenologically favorable for the neutrino Yukawa matrix to be nearly diagonal at low energy. Suppose that the neutrino Yukawa couplings do not have a non-trivial fixed point with some reasons. Then these couplings are not aligned to the democratic form and become dependent on their initial couplings at the Planck scale. Also we may find a sizable parameter region for the initial couplings consistent with the observed neutrino mass and mixings.

In practice such a situation turns out to be realized with a simple assumption, since the right-handed neutrino is a singlet of the $SU(5)$. In general, GUT models may contain a pair of superheavy vector-like fields and so on. Then the right-handed neutrino is allowed to have additional Yukawa interactions with them [10]. These Yukawa couplings are driven to be very large like others by the strong gauge interactions. Consequently anomalous dimension of the right-handed neutrino is enhanced significantly. We note that the IR fixed point stands upon balance of negative contribution by gauge interactions and positive contribution by Yukawa interactions. Presence of extra Yukawa couplings may destroy this balance and take the IR fixed point away from the neutrino Yukawa couplings in the end. Explicitly the coefficient $c$ in the fixed point equation (29) is made negative by the

[9] In the case with large $\tan\beta$, we have to take into account large SUSY threshold corrections on the bottom mass[24].

[10] R-parities for the extra fields may be assigned properly.

additional radiative corrections. Thus the remarkable difference in the mixing matrices of quark and lepton sectors may attribute to the dynamics of the right-handed neutrino.

We shall demonstrate the above mechanism by examining the RG equations only for the neutrino Yukawa couplings. As a toy model with a single flavor, let us consider the superpotential given by

$$W = Y^{\nu}\ \bar{\mathbf{5}}\ \mathbf{1}\ H(\mathbf{5}) + \kappa \mathbf{1}\ \Phi\ \bar{\Phi}, \tag{63}$$

where the extra matter $\Phi$ and $\bar{\Phi}$ belong to $SU(5)$ representations $\mathbf{R}$ and $\bar{\mathbf{R}}$ respectively. The RG equations for $\alpha_y = |Y^\nu|^2/8\pi^2$, and $\alpha_\kappa = |\kappa|^2/8\pi^2$ are found to be

$$\mu\frac{d\alpha_y}{d\mu} = \left[7\alpha_y + R\alpha_\kappa - 4C_2(\mathbf{5})\alpha_g\right]\alpha_y, \tag{64}$$

$$\mu\frac{d\alpha_\kappa}{d\mu} = \left[5\alpha_y + (R+2)\alpha_\kappa - 4C_2(\mathbf{R})\alpha_g\right]\alpha_\kappa, \tag{65}$$

where $C_2$ denotes the Casimir index of each representation. Also these are rewritten into equations in terms of $x_y = \alpha_y/\alpha_g$ and $x_\kappa = \alpha_\kappa/\alpha_g$ as

$$\mu\frac{dx_y}{d\mu} = \left[7x_y + Rx_\kappa - 4C_2(\mathbf{5}) + b\right]\alpha_g x_y, \tag{66}$$

$$\mu\frac{dx_\kappa}{d\mu} = \left[5x_y + (R+2)x_\kappa - 4C_2(\mathbf{R}) + b\right]\alpha_g x_\kappa. \tag{67}$$

There are four fixed points in the coupling space of $(x_y, x_\kappa)$, three of which are immediately seen from the above equations as

$$(x_y, x_\kappa) = (0,0), \tag{68}$$

$$= \left(\frac{4C_2(\mathbf{5}) - b}{7}, 0\right), \tag{69}$$

$$= \left(0, \frac{4C_2(\mathbf{R}) - b}{R+2}\right). \tag{70}$$

The first two fixed points are what we have been discussing so far and the third one is a new entry. It is noted that the fourth one appears in the region of negative $\alpha_y$ for a large representation $\mathbf{R}$ [11]. Here we pay attention to such a case.

The RG flow diagram for $(x_y, x_\kappa)$ is shown in Fig. 2 in the case of $b = 0$ and $\mathbf{R} = \mathbf{adj}$ as an example. The marked points A, B and C stands for the fixed points given by (68), (69) and (70) respectively. It is seen that the fixed point B is now unstable towards direction of the new coupling, while C turns out to be IR attractive instead. Thus the neutrino Yukawa coupling is found to decrease at low energy in the presence of $\kappa$.

In Fig. 3 evolution of neutrino Yukawa coupling with respect to the scale is shown. Each line corresponds with a RG flow presented in Fig. 2. It is found that the Yukawa

[11] This point may not be unphysical with taking complex phases of the Yukawa coupling into considerations. However this fixed point is not IR attractive anyway.

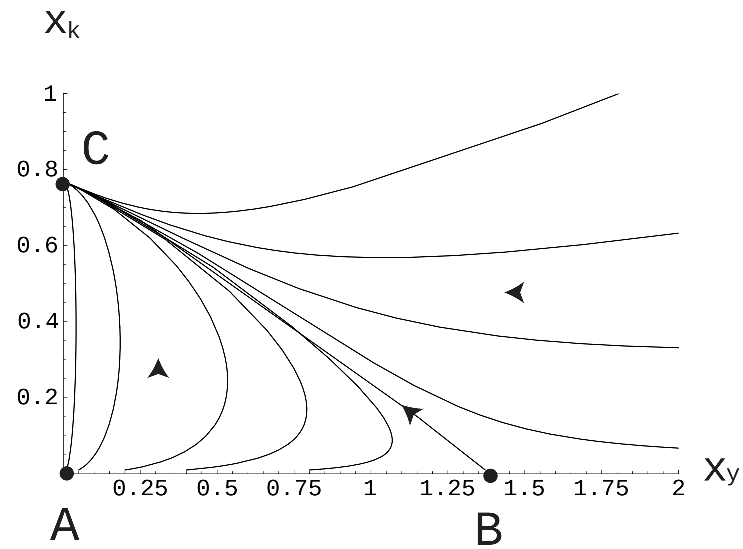


Figure 2: RG flows for $(x_y, x_\kappa)$ in the case of $b = 0$ and $\mathbf{R} = \mathbf{adj}$. The arrows show the direction to lower energy scale. The circles at A, B and C stand for the fixed points given by (68), (68) and (70) respectively.

couplings decrease monotonically at low energy and are not aligned. It is expected that the neutrino Yukawa couplings for multi-flavors also show the same aspect of these flows, even though other Yukawa couplings $Y^u$ and $Y^d$ are aligned to the fixed point. We now add the following terms,

$$Y^\nu_{ij}\bar{\mathbf{5}}_i\mathbf{1}_j H(\mathbf{5})_{ij} + \kappa_i \mathbf{1}_i \Phi\ \bar{\Phi} + M_{Rij}\mathbf{1}_i\mathbf{1}_j, \tag{71}$$

to the superpotential given by Eq. (49). It is easily found that ratio of the neutrino Yukawa couplings follows the RG equation given by

$$\begin{aligned} \mu\frac{d}{d\mu}\ln\left(\frac{\alpha_{y^\nu_{ik}}}{\alpha_{y^\nu_{jk}}}\right) &= \left(\gamma_{\mathbf{5}^*_i} + \gamma_{\mathbf{1}_k} + \gamma_{H(\mathbf{5})_{ik}}\right) - \left(\gamma_{\mathbf{5}^*_j} + \gamma_{\mathbf{1}_k} + \gamma_{H(\mathbf{5})_{jk}}\right) \\ &= a_{\bar{\mathbf{5}}}\left(\sum_k \alpha_{y^\nu_{ik}} - \alpha_{y^\nu_{jk}}\right) + 3a_H\left(\alpha_{y^\nu_{ik}} - \alpha_{y^\nu_{jk}}\right), \end{aligned} \tag{72}$$

where the Yukawa couplings other than $\alpha_y{}^\nu$ are assumed to be their fixed point values. Since the neutrino Yukawa couplings are suppressed as seen above, the right-hand side of (72) becomes very small quickly. This shows that ratio of the neutrino couplings is almost unchanged at low energy region.

In the Frogatt-Nielsen mechanism [6], the neutrino Yukawa couplings are constrained upto O(1) coefficient, and, therefore, the masses and mixing angles for neutrinos are not predicted. However we may find a sizable probability for the couplings distributed in anarchy to be compatible with the observed masses and mixing angles [25].

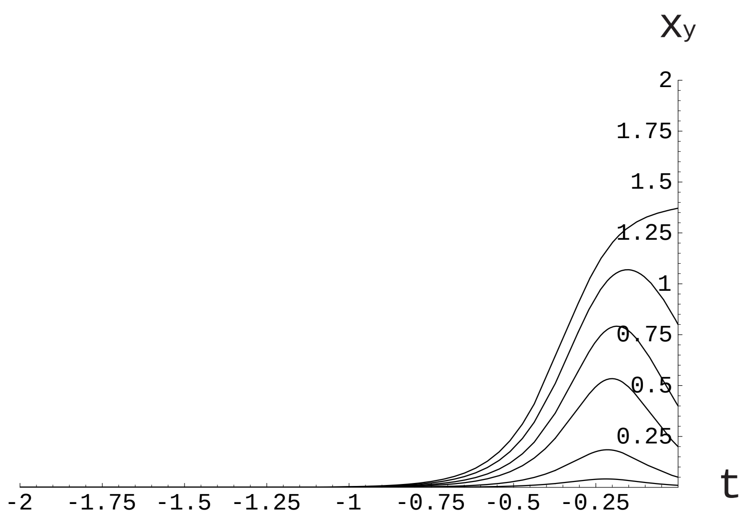


Figure 3: Running of $x_y$ corresponding with various flow lines shown in Fig. 2 with respect to the scale $\mu$ ($t = \ln(\mu/M_{\rm pl})$).

Now if three of nine initial neutrino Yukawa couplings happen to be larger than others to some extent, then they may be dominant in the low energy matrix also. Since there is no specific flavor basis, we may regard these couplings as the diagonal elements. Thus it is not so special that the neutrino Yukawa coupling matrix appears close to diagonal, and therefore the large lepton mixing angles are described well. We leave explicit survey of such a parameter space for future works.

In our mechanism the neutrino Yukawa couplings are largely suppressed as seen in Fig. 3. However this does not imply that the see-saw mechanism offers us very tiny neutrino masses. Note that this suppression occurs because of enhancement of anomalous dimensions of the right-handed neutrinos. Then their Majorana masses $M_R$ are also suppressed by the anomalous dimensions. Consequently it is found that the neutrino masses themselves are not suppressed through this mechanism. It is noted that the masses for the right-handed neutrinos are expected to appear at the intermediate scale in our scenario, even though the bare parameter for $M_R$ is set to be the GUT scale.

# 6 Setup for the realistic mass matrices

How can we describe mass differences among the first and the second generations at all? In the phenomenological approach, the two flavor symmetry breaking parameters $\epsilon$ and $\delta$ are taken to be hierarchical as given in Eq. (6). Besides it is necessary for these breaking parameters to be diagonal elements, which is not explained from the standpoint of flavor symmetry either.

Now the problem to be considered is how to realize such a specific form of deviations

from the democratic couplings in our dynamical framework. The first possibility is to generate such deviations away from the fixed point by assuming proper initial couplings. We consider this by examining the RG equations given by Eq. (29) again. Suppose, *e.g.* that one of the Yukawa couplings, say $Y_{33}$, happens to be fairly smaller than the others at the Planck scale. Then $x_{ij}$ for $i, j = 1, 2$ converge into the fixed point value rapidly, since the beta functions for these couplings do not contain $x_{33}$. The couplings $x_{i3}$ and $x_{3i}$ for $i = 1, 2$ may be affected by $x_{33}$. So we may examine the RG flows with neglecting differences among $x_{ij}$ for $i, j = 1, 2$ in the first step of approximation. In this situation the deviations from the fixed point $\Delta x_{ij} = x_{ij} - x^*$ are found to satisfy

$$\mu\frac{d}{d\mu}\left[\Delta x_{13} - \Delta x_{23}\right] = \alpha_g x^*(a' - a_u)\left[\Delta x_{13} - \Delta x_{23}\right] \tag{73}$$

$$\mu\frac{d}{d\mu}\left[\Delta x_{31} - \Delta x_{32}\right] = \alpha_g x^*(a' - a_Q)\left[\Delta x_{31} - \Delta x_{32}\right]. \tag{74}$$

These equations tell us that these differences shrink rapidly with scaling down and eventually the couplings satisfy the relations

$$x_{13} = x_{23}, \quad x_{31} = x_{32}. \tag{75}$$

Moreover differences among $x_{ij}$ for $i, j = 1, 2$, which is ignored in the above argument, are found to satisfy

$$\mu\frac{d}{d\mu}\left[\Delta x_{11} - \Delta x_{21}\right] = \mu\frac{d}{d\mu}\left[\Delta x_{12} - \Delta x_{22}\right] = \alpha_g x^* a_Q\left[\Delta x_{13} - \Delta x_{23}\right], \tag{76}$$

$$\mu\frac{d}{d\mu}\left[\Delta x_{11} - \Delta x_{12}\right] = \mu\frac{d}{d\mu}\left[\Delta x_{21} - \Delta x_{22}\right] = \alpha_g x^* a_u\left[\Delta x_{31} - \Delta x_{32}\right]. \tag{77}$$

Therefore it is seen that these differences are not affected from large deviation of $x_{33}$ and are kept small. In Fig. 4 a sample of RG flows for the couplings $x_{ij}$ obtained by numerical analysis are demonstrated. The bold line shows flow of $x_{33}$, whose initial coupling is assumed to be relatively smaller than others. Note that Yukawa coupling $Y_{33}$ itself is not so apart from others. The dashed lines represent flow of $x_{i3}$ and $x_{3i}$ for $i = 1, 2$, while the dotted lines represent flow of $x_{ij}$ for $i, j = 1, 2$. Indeed behavior of these RG flows supports the above argument.

Similar forms of Yukawa coupling matrices may be obtained by introducing a weak interaction breaking the flavor democracy. For example, we assume only one of the Higgs fields, say $H_{33}$, has an extra superpotential $\lambda H_{33}\Phi_1\Phi_2$ with very small coupling $\lambda$. We consider the linear perturbation around the fixed point. The extra Yukawa interaction affects only on the beta function of $\Delta x_{33}$. To be explicit, the RG equations given in Eq. (34) are modified to

$$\mu\frac{d\Delta x_{ij}}{d\mu} = \alpha_g x^*\left[(\mathcal{M}\Delta x)_{ij} + \delta_\lambda \delta_{i3}\delta_{j3}\right], \tag{78}$$

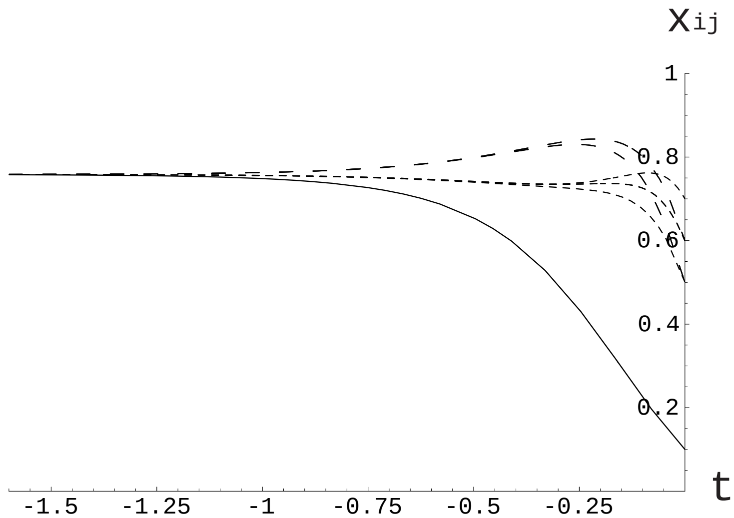


Figure 4: Convergence behavior of $x_{ij}$ to the IR fixed point in the case of a proper initial couplings. The parameter $t$ denotes $\ln(\mu/M_{\rm pl})$. The bold line shows flow of $x_{33}$, whose initial coupling is assumed to be rather smaller than others. The dashed lines represent flow of $x_{i3}$ and $x_{3i}$ for $i = 1, 2$, while the dotted lines represent flow of $x_{ij}$ for $i, j = 1, 2$.

where $\mathcal{M}$ denotes the matrix given in Eq. (34) and $\delta_\lambda \alpha_g x^* \sim |\lambda|^2/8\pi^2$. When differences among the initial Yukawa couplings happen to be rather small already, the same equations given by Eqs. (73), (74), (76) and (77) hold as well.

In the end, the Yukawa couplings turn out to be represented by the form of

$$Y = y_0 \left( \begin{array}{ccc} 1+O(\epsilon) & 1+O(\epsilon) & 1+\delta' \\ 1+O(\epsilon) & 1+O(\epsilon) & 1+\delta' \\ 1+\delta'' & 1+\delta'' & 1+\delta \end{array} \right), \tag{79}$$

at low energy scale. Here $\epsilon$ stands for the incompleteness of strong convergence to the fixed point, which can be as small as $10^{-5}$. Other parameters representing deviations from the fixed point $\delta$, $\delta'$ and $\delta''$ are supposed to be of the same order. Though these values are dependent on the initial Yukawa couplings, we may well assume $\epsilon \ll \delta \ll 1$. After transforming this matrix $Y$ by using the diagonalizing matrix given by Eq. (2), we obtain

$$A^T Y A = y_0 \left( \begin{array}{ccc} O(\epsilon) & O(\epsilon) & O(\delta) \\ O(\epsilon) & O(\delta) & O(\delta) \\ O(\delta) & O(\delta) & 3+O(\delta) \end{array} \right). \tag{80}$$

This is very similar to the Fritzsch type matrix and, therefore, offers us a phenomenologically favorable pattern for realistic masses and mixing angles. The ratio mass eigenvalues is given by $O(\epsilon) : O(\delta) : O(1)$. We do not study which initial couplings really give the viable mass matrices explicitly, since there are large model dependence.

# 7 Conclusions and discussions

The approach of democratic mass matrix seems to be attractive enough phenomenologically. The notion of flavor democracy offers us an interesting viewpoint distinct from other approach. Above all it is noted that the lepton mixing angles, bi-large mixings and small $U_{e3}$, are well described by assuming a democratic mass matrix for the charged leptons.

However it is also true that these phenomenologically favorable matrices are not explained by flavor symmetries. In this paper we considered the models in which the democratic type of Yukawa couplings are realized as an IR attractive fixed point[18]. For this purpose we introduced multi Higgs fields, from which only one massless mode is allowed by a special type of mass terms. The large mass hierarchies are achieved easily by using strong gauge interaction for quarks and leptons as well as the Higgs fields. To be explicit, we considered an $SU(5) \times SU(5)$ GUT model, in which one of the gauge couplings is non-perturbatively strong.

Indeed the setup of the Higgs fields may be somewhat artificial. However we found some benefits in this scenario. The democratic Yukawa couplings are realized even in GUT models without any adhoc assumptions. Hence small mixing angles as well as large mass hierarchies in the quark sector are naturally obtained. Especially the large lepton mixing angles can be explained through extra interactions of the right-handed neutrinos. It is seen that simple setting of the initial couplings may lead to low energy Yukawa couplings of the Fritzsch type. Also this kind of scenarios predict top quark mass in the good agreement with experiment and large $\tan\beta$.

In this article we considered only supersymmetric models. However the PR fixed point exists also in non-supersymmetric theories as well. Therefore the democratic mass matrices may be realized also in some non-supersymmetric models, though the beta functions for Yukawa couplings are complicated.

In section 3 we mentioned that the PR fixed point appears also in extra dimensions and shows rather strong convergence in general. However naive extension with gauge and Higgs in the bulk of the extra dimensions does not work. This is because anomalous dimensions of the Higgs fields vanish due to effective N=2 supersymmetry, and ,therefore, the democratic type of fixed point is not realized. This would not deny any possibility for extra dimensional models, and we leave it for future study.

Lastly some comments on soft supersymmetry breaking parameters would be in order. [12] It has been known [27, 28] that the PR fixed point induces peculiar relations among soft parameters also. Above all A-parameters are found to be aligned in the present scenarios with the same dynamics for the Yukawa coupling alignment. The soft scalar masses for quarks and leptons become flavor universal at low energy due to large corrections by the democratic Yukawa couplings as well as the gauge couplings. These properties are very desirable for the flavor problem in supersymmetric extensions.

However the strong gauge dynamics enhances the soft scalar masses and also the

[12]The various flavor violating processes have been examined in the phenomenological approach of the democratic mass matrices[26].

A-parameters to be comparable with the strongly coupled gaugino mass. Therefore the gaugino mass in the strongly coupled sector must be much smaller than the MSSM gaugino masses. In this respect, extension to extra dimensions will be considerable, since FCNC and also CP problems may be solved or ameliorated without assuming such a special case [29]. Otherwise it would be more natural for our scenarios that supersymmetry breakings are mediated below the GUT scale *e.g.* by gauge mediation mechanism. Further studies of the supersymmetry breaking parameters in our models will be discussed elsewhere.

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